Multistability and soliton modes in nonlinear microwave resonators

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Nonlinear optical cavities are known to exhibit bistability and multistability. In this letter the properties of microwave Fabry–Perot and ring resonators with second-order dispersive nonlinearity are analyzed. Experimental results such as multiple-valued output versus input characteristics, resonance curves, and waveforms are described. On the basis of harmonic and subharmonic generation a new mechanism is found leading to multistability, which is quantitatively attributed to well-defined soliton modes propagating in the resonator.

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Bistable optical devices where nonlinearity and feedback lead to two distinct stable states of transmission have recently attracted much attention.[1] Besides, systems are now in consideration exhibiting multistability in case of sufficiently large nonlinearities, leading to more than two stable states for the same setting of parameters. Marburger and Felber[2] derived that a Fabry–Perot resonator with a nonlinear Kerr medium will show a multiple-valued output versus input intensity characteristic. In case of a ring resonator containing a two-level absorber, Ikeda[3] predicted a similar behavior. To a certain extent, multistability has now been confirmed by some experimental situations: Okada and Takizawa[4] succeeded in multiple-valued operation using a hybrid electro-optical device. With the aid of pure intrinsic optical devices, Miller *et al.*[5] and Eichler[6] observed a steplike multistable behavior. Beyond that, Gozzini *et al.*[7] investigated the multistable response of a Fabry–Perot microwave resonator excited by several electromagnetic waves. In the present paper the multistability of microwave Fabry–Perot and ring transmission line resonators[8,9] is examined. The second-order nonlinear material under investigation has the properties of a Korteweg–de Vries (KdV) system.[10]

In a first exemplary experiment the frequency f of the sinusoidal input signal corresponds to the fifth natural mode of the resonator ($N = 5$). As a first result, the transmission curve in Fig. 1 exhibits multistability with six clearly different states of output power. A further inspection, however, reveals that two of them consist of two different, barely separated states. As a result of this "degeneracy," one can ultimately distinguish between eight different states of transmission, and by supplying a fixed input power, the output power may achieve up to six values in the present case.

In order to examine the behavior of the resonator in detail, in a second step the spectrum of the transmitted wave has been analyzed. At relatively low input powers the observed spectrum can essentially be determined by the generated second harmonic (cf. branch No. 1 in Fig. 1). On increasing the input power, successive jumps into higher transmission branches occur where in addition to the harmonics nf (n natural number), subharmonics nf/N are also generated.[11] Thus, in the present case the spectrum is completely described by Fourier components that are multiples of $f/5$ up to the cut-off frequency of the material. However, it turns out to be difficult to distingush between the different observable states by means of the individual Fourier components only. Nevertheless, a more physical concept can be introduced as follows allowing a much better description of the background of multistability. For that purpose from here on we consider the temporal profile of the wave propagating in the resonator. In order to exclude additional spatial phenomena within the Fabry–Perot and to illustrate the idea the following discussion is restricted to typical measurements performed on the ring resonator.

From the theoretical viewpoint, in our device the stationary periodic and soliton solutions of the KdV equation are known to be of remarkable importance.[9,10,12] Moreover, regarding the impressed boundary conditions of the resonator, the behavior of such a device can be attributed to a model as described in Ref. 13 where solitons interact with the harmonic pump wave synchronously moving in the same direction.

As an experimental example a particular waveform is illustrated in Fig. 2(a).[14] As can be seen the first of five periods of the pump contains a single pulse which reveals to be a KdV soliton. This soliton is parametrically generated by and superimposed upon the sinusoidal pump wave. Since the entire period is five times that of the pump just one soliton is moving in the ring. Thus this mode can be characterized by the bit pattern (10000). Another branch of the transfer curve is characterized by the waveform of Fig. 2(b). Now three solitons are propagating in the ring. The arrangement corresponds to (11010). To complete the description, Table I lists

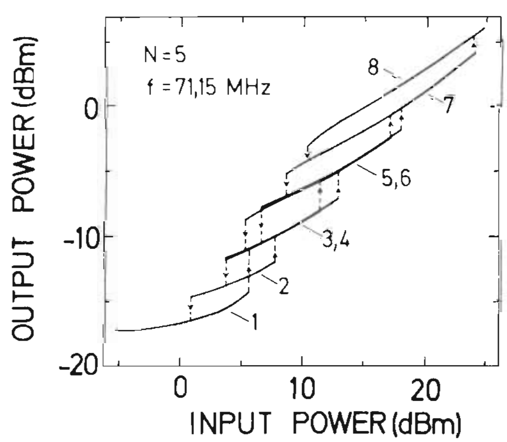

FIG. 1. Multistability; experimental dependence of the transmitted power on the power incident on the Fabry–Perot resonator. $f \simeq 5 f_1$, where $f_1 = 14.1$ MHz denotes the fundamental eigenfrequency.

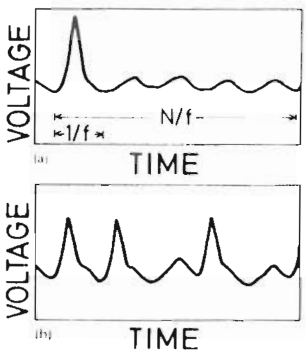

FIG. 2. Stationary waveform as measured in the ring resonator with $N = 5$. The fundamental period is N/f which equals one round trip time. The period of the pump is $1/f$. (a) State No. 2; (b) State No. 6, cf. Figs. 1 and 3 and Table I.

the eight sequences of all observed states in case $N = 5$. From Table I one also concludes that the above degeneracy of states (cf. Fig. 1) is due to the same number of solitons but with different arrangements (modes 3 and 4, 5, and 6). In general, if N denotes the number of possible resonator modes, a soliton may be in any of N discrete positions relative to a reference period which is N times that of the pump. With respect to the periodicity of the problem and applying methods from combinatorial analysis and number theory the maximal number A of states can be calculated in the following manner. If d represents the set of all natural divisors of N and q in turn that of d, then A can be written as

$$A(N) = \sum_d \frac{F(d)}{d}, \tag{1}$$

where

$$F(d) = \sum_q u(q)\, 2^{d/q} \tag{2}$$

is the Möbius inversion formula.[15] If r is the number of prime factors decomposing q, then the Möbius function $u(q)$ is defined as follows: $u(1) = 1$, $u(q > 1) = (-1)^r$ provided that the prime decomposition is square-free, and $u(q) = 0$ otherwise. Taking an example, $N = 8$ yields $d = 1; 2; 4; 8$ and the corresponding values of q are given by 1; 1, 2; 1, 2, 4; 1, 2, 4, 8. The values $q = 1, 2, 4, 8$ result in $u = 1, -1, 0, 0$. Thus one obtains $A(8) = 36$; by setting $N = 20$ there are already 52 488 distinct states. Experimentally the formula has been verified up to $N = 8$.

TABLE I. Soliton modes: arrangement of solitons relative to the pump ($N = 5$; fifth natural mode of the resonator). Each column corresponds to five periods of the pump. The state "one soliton" is marked by "1," the state "no soliton" is marked by "0."

Pump period \ Mode	1	2	3	4	5	6	7	8
1st	0	1	1	1	1	1	1	1
2nd	0	0	1	0	1	1	1	1
3rd	0	0	0	1	1	0	1	1
4th	0	0	0	0	0	1	1	1
5th	0	0	0	0	0	0	0	1

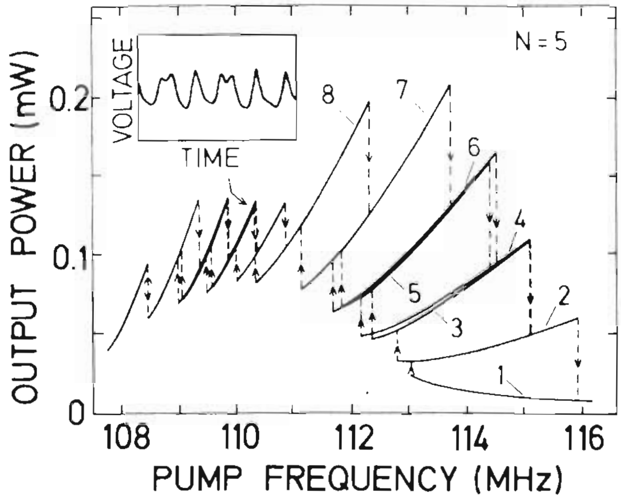

FIG. 3. Nonlinear resonance curve of the ring resonator, input power is 0.325 mW. The inset displays the wave profile wihtin the state as marked by an arrow.

Finally, we demonstrate in Fig. 3 that the nonlinear resonance curves also exhibit a distinct multistable behavior. Here the hysteresis cycles and states of transmission within the higher frequency range ($f > 111$ MHz) are equivalent to those of the transfer curve in Fig. 1. The corresponding waveforms are those listed in Table I. Moreover, additional hysteresis cycles can be discovered at lower frequencies ($f < 111$ MHz). The typical features of the propagating waves belonging to this frequency domain can be judged from the inset of Fig. 3. As a result, parts of the series consist of single solitons per period of the pump, but there are also definite positions with two pulses per period. On the other side, this observation can be traced back to a pronounced resonant behavior of the generated second harmonic, which gets a large amplitude and in turn produces extra resonances by nonlinear interactions, cf. Ref. 8. Namely, due to the normal dispersion of the transmission line, in the nonlinear system the second harmonic is in resonance at pump frequencies which are smaller than the resonance frequency (in the linear regime this behavior implies that the resonance frequency in case of $N = 10$ is smaller than twice that of $N = 5$).

In conclusion, we have shown that multistability can be observed by using Fabry–Perot and ring resonators with a second-order dispersive nonlinearity where in contrast to the field of nonlinear optics multistability occurs in the vicinity to a single resonance. Thus, this mechanism permits multistable operations without the enormous input powers necessary in the optical case to switch into higher states. Together, it should be mentioned that the transfer curves display not only a steplike behavior, but also a "real" multistability at specified settings of all parameters of the input wave.

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